Sub-Riemannian days 2017
Institut Fourier, Grenoble

17-18 October 2017

Deformation of singular spaces

Nicolas JUILLET (Université Strasbourg)

Gigli and Mantegazza have observed how optimal transport and heat diffusion allow to describe the direction of the Ricci flow uniquely from the metric aspects of Riemannian manifolds. Their goal is to reformulate the Ricci flow so that it also makes sense for metric spaces. I will present investigations and results obtained with Matthias Erbar (univ. Bonn) that concerns some non-Riemannian limits of Riemannian manifolds, in particular the Heisenberg group.

Excess and tangents of sub-Riemannian geodesics

Roberto MONTI (Universita Padova)

We present some recent results on the regularity problem of sub-Riemannian length minimizing curves. This is a joint work with A. Pigati and D. Vittone. After introducing the notion of excess for a horizontal curve, we show that at any point of a length minimizing curve excess is infinitesimal at some sequence of scales. This implies the existence of a linear tangent. We also discuss other results related to excess.

Poincaré inequalities for differential forms on Heisenberg group

Pierre PANSU (Université Paris Sud)

Every closed differential form ω on a Euclidean ball has a primitive whose L^q norm is bounded by the L^p norm of ω (for suitable exponents p and q). We prove an analogous result for Rumin's exterior differential on Heisenberg balls. This is used to prove vanishing of $\ell^{q,p}$-cohomology of Heisenberg groups. Joint work with Annalisa Baldi and Bruno Franchi.

Besicovitch covering property in sub-Riemannian geometry

Severine RIGOT (Université Nice)

The Besicovitch covering property originates from works of Besicovitch about differentiation of measures in Euclidean spaces. It can more generally be used as a usefull tool to deduce global properties of a metric space from local ones. We will discuss in this talk the validity or non validity of the Besicovitch covering property on stratified groups equipped with sub-Riemannian distances (Carnot groups) and more generally on graded groups equipped with homogeneous distances. We will illustrate these results with explicit examples in the Heisenberg group. We will also discuss some consequences related to the theory of differentiation of measures on sub-Riemannian manifolds. Based on joint works with E. Le Donne and S. Nicolussi Golo.

Optimal Control, Differential Games, Mean Field Games, and Pontryagin and Hamilton-Jacobi equations on probabilities

Filippo SANTAMBROGIO (Université Paris Sud)

The talk will be a short introduction to the emerging topic of Mean Field Games in connection with optimal control and differential games. I will present what is in general a Mean-Field Game and how to translate it into a coupled system of a forward continuity (or Fokker-Planck) equation on the density of players and of a backward Hamilton-Jacobi equation on their value function. Then I will focus on the case where this system has a variational origin and I will explain how this system actually corresponds to Pontryagin's maximum principle in the space of measures... wait, wait : if thys system, which includes a Hamilton-Jacobi equation, is Pontryagin, what is the Hamilton-Jacobi equation in this case? the talk will finish evoking some answers to this question, for control problem in the space of measures and also for differential games, thus arriving up to the so-called "Master equation" introduced by P.-L. Lions, which will be sketched.

Whitney C^1 extension of horizontal curves in sub-Riemannian manifolds

Mario SIGALOTTI (Ecole Polytechnique)

We discuss the validity of the Whitney C^1 extension property for horizontal curves endowed with 1-jets that satisfy a first-order Taylor expansion compatibility condition. We start by recalling some recent results obtained for Carnot groups, where a complete characterization of the extension property is known in terms of the openness of some suitable input-output maps. We then consider the equiregular sub-Riemannian case,

where we show that the extension property holds true whenever some non-singularity property holds for the input-output maps on the Carnot groups obtained by nilpotent approximation. We conclude by discussing the case of singular sub-Riemannian manifolds.